

# Financial globalization and economic growth

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## Abstract

We analyze the effects of capital mobility on the speed of convergence and on growth. We compare a closed and an open economy, using a two-sector endogenous growth model with adjustment costs. Capital mobility leads to a smaller speed of convergence and growth in the open economy than in the closed economy. In the closed economy, the remuneration of human capital is decreasing during the transition, while the remuneration of physical capital is increasing. In the open economy, human capital is nontradable and the alternative investment yields the international interest rate, corresponding to the remuneration of physical capital of the closed economy in the balanced growth path. The no-arbitrage condition shows a larger difference in the remuneration of the two capitals in the closed economy. It leads to a higher accumulation of human capital and thus to a faster speed of convergence in the closed economy. This result reverses the conclusion given by the neoclassical growth model for the same experiment.

**JEL Classification:** F43, O41

**Keywords:** capital mobility; speed of convergence; growth; open versus closed economies

# 1 Introduction

Financial globalization has increased since 1970, with a more pronounced path after the mid-eighties<sup>1</sup>. Following this path, there is a growing literature on the empirical effects of financial openness on growth. In surveys of the studies on this issue, Kose et al. (2006), Henry (2007) and Obstfeld (2008) conclude that there is no robust and positive effects of international financial integration on growth. This is a surprising conclusion, considering the implications of the one sector neoclassical growth model. In fact, one of the expected gains from financial globalization are the positive effects on convergence and growth.

In this paper we want to analyze the effects of capital mobility in the speed of convergence and thus on growth. Departing from a two-sector endogenous growth model, à la Lucas (1988) and including adjustment costs, we will show that the speed of convergence of the open economy is smaller than that of the closed economy. Consequently, international financial integration would lead to a reduction on growth during the transition. This result reverses the conclusion given by the one sector neoclassical growth model for the same experiment. Using the neoclassical model, Gourinchas and Jeanne (2006) show the positive effects of capital mobility on convergence and growth, and point out that the welfare gains of capital mobility are small.

On the one hand, we suppose a closed economy that initially has relatively more physical capital than human capital (or infra-structures)<sup>2</sup>. The remuneration of human capital is relatively higher than that of physical capital and there is an incentive to invest relatively more in human capital. As human capital becomes relatively more available, its relative price (the price of human capital over the price of physical capital) decreases. During the

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<sup>1</sup>These patterns are presented in Kose, Prasad, Rogoff and Wei (2006) using two proxies for financial openness. See Figure 3 in their paper. A de jure proxy measures restrictions on capital account and equals one if there are no restrictions. The data are from the IMF's Annual Report on Exchange Arrangements and Exchange Restrictions. A de facto proxy is based on the ratio of gross stocks of foreign assets and liabilities to GDP of a country. It is based on Lane and Milesi-Ferretti (2007).

<sup>2</sup>We would like to point out that the analysis would be the same if there was relatively more human capital available.

transition to the balanced growth path, the remuneration of human capital (whose production is more intensive in human capital) will be decreasing while the remuneration of physical capital will be increasing, until we arrive to the balanced growth path investment rate. Then we can analyze the transitional dynamics using a no-arbitrage condition, where the difference in the remuneration of both capitals gives the incentive to accumulate relatively more one of the capitals. The no-arbitrage condition provides the transitional dynamics of the model in a simple way and gives an intuition of the mechanism at work.

On the other hand, we consider a small open economy with the same structure of production of the closed economy. Now consumption goods and physical capital are tradable and human capital is nontradable. The convergence of the economy can be analyzed again with the no-arbitrage condition, where now the alternative to investing in human capital is a foreign bond yielding the international interest rate. As for the closed economy, suppose that at time zero physical capital is relatively more abundant than human capital. During the transition, the remuneration of human capital will be higher than the international interest rate and the latter higher than the remuneration of physical capital. It follows that a no-arbitrage condition gives an incentive to invest relatively more in human capital, but this incentive is higher in the closed economy than in the open economy. The higher incentive in the closed economy to invest relatively more in human capital also leads to a faster speed of convergence in the closed economy than in the open economy.

We analytically derive all the results for the Lucas (1988) model with adjustment costs. In addition, we show that our main result applies for any finite value of the adjustment costs, in an economy with incomplete specialization. Then we generalize the model, including physical capital in the production of human capital. We show the robustness of the main result of the paper with the help of a numerical example, and using values for the adjustment costs in line with the literature.

Our paper is related to several important strands of theory on growth, convergence and openness. First there is the literature on the two-sector

endogenous growth models, as Lucas (1988) and Rebelo (1991). The characterization of transitional dynamics of these models is seen in Bond, Wang and Yip (1996), while Mulligan and Sala-i-Martin (1993) provide numerical simulations. Ortigueira and Santos (1997) arrive at an analytical expression for the speed of convergence of a two-sector endogenous growth model based on a theorem presented in the appendix of their paper. However, they do not derive an expression for the speed of convergence of a closed economy with adjustment costs, as we do. In our paper, we also present an intuition for the transitional dynamics based on a no-arbitrage condition. A second strand, with tradition in international economics, makes a distinction between tradable and nontradable goods. For one of the first formalization in the spirit of our model see Bruno (1976). Farmer and Lahiri (2006) analyze the transitional dynamics of a two-country world economy. These authors use a two-sector endogenous growth model and make a distinction between tradable and nontradable sectors, however they do not present analytical expressions for the speed of convergence. Turnovsky (1996) develops a dependent economy using a two-sector endogenous growth model, but he is not analyzing the speed of convergence. He is interested in the dynamics of the model following a shock in the parameters.

The paper is organized as follows. Section 2 presents a two-sector endogenous growth model with adjustment costs. Section 3 develops a closed economy version with adjustment costs and its dynamics. In Section 4 we present a small open economy with adjustment costs and its dynamics. In Section 5 we generalize the basic model and we present a numerical example of our main result. Concluding remarks are presented in Section 6. The Appendix contains details of the derivation of some analytical results.

## 2 The model

This section presents the structure of a two-sector endogenous growth model with adjustment costs for physical capital.

An analysis of the general model (where the production of human capital also uses physical capital, but without adjustment costs) and its dynamics

is made by Bond et al. (1996), who present an analytical and geometrical solution to the dynamics of the model<sup>3</sup>.

The utility function of the representative agent is given by:

$$U(C) = \log C \tag{1}$$

where  $C$  represents consumption.

There are two sectors of production, one ( $Y^K$ ) for consumption goods and accumulation of physical capital,  $\dot{K}$ , and the other ( $Y^H$ ) for accumulation of human capital,  $\dot{H}$ . The technologies are similar to the ones in Lucas (1988), but there are no externalities.

We have

$$Y^K = AK^\alpha (uH)^{1-\alpha}, \tag{2}$$

$$Y^H = B[(1-u)H]. \tag{3}$$

Without loss of generality, the depreciation rate is assumed equal to zero for physical capital and human capital.  $\alpha$  ( $0 < \alpha < 1$ ) is a parameter with  $1 - \alpha$  representing the share of nontradable capital in the consumption sector and  $u$  ( $0 \leq u \leq 1$ ) is the fraction of the human capital used in the consumption sector.  $A$  and  $B$  represent the level of technology in the consumption and the human capital sectors, respectively.

Using the structure of production as in Lucas (1988), we are assuming that the production of goods and physical capital is more intensive in physical capital. This leads to the stability of the differential equation associated with the relative price, as shown in Bond et al. (1996).

The planning problem for this (closed) economy is given by

$$\max_{C,u,I,K,H} \int_0^{+\infty} (\log C) e^{-\rho t} dt \tag{4}$$

subject to

$$AK^\alpha (uH)^{1-\alpha} = C + I \left[ 1 + m \left( \frac{I}{K} \right) \right] \tag{5a}$$

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<sup>3</sup>Mulligan and Sala-i-Martin (1993) study the transitional dynamics giving values to the parameters of the model.

$$\dot{K} = I \quad (5b)$$

$$\dot{H} = B[(1 - u)H]. \quad (5c)$$

$K_0 > 0$  and  $H_0 > 0$  are given and  $C \geq 0$ . The parameter  $\rho$  represents the subjective discount rate and  $I$  represents investment in physical capital. The expression of the adjustment costs,  $m\left(\frac{I}{K}\right)$ , is represented in the following way<sup>4</sup>:

$$m\left(\frac{I}{K}\right) = \frac{\frac{h}{2}\left(\frac{I}{K} - a\right)^2}{\frac{I}{K}}. \quad (6)$$

$$a = g$$

$$g = (r^K)^* - \rho > 0 \quad (7)$$

The parameter  $h$  is the sensitivity of the adjustment costs to the ratio investment to physical capital. The value of the Tobin's  $q$  in the balanced growth path will be  $q^* = 1$ , as we assume  $a = g$ , and  $g$  represents the growth rate of the closed economy in the balanced growth path. This growth rate will also be equal to the growth rate of the open economy in the balanced growth path. And  $(r^K)^* = r^*$  gives the marginal product of physical capital and the real interest rate of the closed economy in the balanced growth path. This real interest rate of the closed economy is also equal to real interest rate of the open economy.

As we are using the same economic structure and all relevant variables converge to the same balanced growth path in both economies, we guarantee the comparability between the closed and the open economies.

## 3 The closed economy

### 3.1 The model and the transitional dynamics

In this section, we provide an intuition of the behaviour of the transitional dynamics, based on a no-arbitrage condition for the accumulation of physical

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<sup>4</sup>Ortigueira and Santos (1997) also use this specification of adjustment costs in an endogenous growth model à la Lucas (1988). See also King and Rebelo (1993) and Summers (1981).

and human capital. This intuition will be the basis for understanding the different behaviour of the speed of convergence of the closed and the open economies.

The Hamiltonian for the problem of the closed economy, given by equations (4) and (5a) to (5c), is

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t} \left\{ AK^\alpha (uH)^{1-\alpha} - C - I \left[ 1 + m \left( \frac{I}{K} \right) \right] \right\} + \\ + \eta q e^{-\rho t} I + \mu e^{-\rho t} \{ B [(1-u)H] \}$$

where  $\eta$  is the Lagrangian multiplier associated with equation (5a),  $\eta q$  is the costate variable in installed physical capital<sup>5</sup> and  $\mu$  is the costate variable in human capital.

We obtain the following first order conditions:

$$C^{-1} = \eta \tag{8a}$$

$$r^K = \alpha AK^{\alpha-1} (uH)^{1-\alpha} \tag{8b}$$

$$r^H = (1-\alpha) AK^\alpha (uH)^{-\alpha} = \frac{\mu}{\eta} B \tag{8c}$$

$$\dot{\mu} = \mu \left( \rho - \frac{\eta}{\mu} r^H \right) \tag{8d}$$

$$q = 1 + h \left( \frac{I}{K} - a \right) \tag{8e}$$

$$\dot{q} = \left( \rho - \frac{\dot{\eta}}{\eta} \right) q - r^K - h \left( \frac{I}{K} - a \right) \frac{I}{K} + \frac{h}{2} \left( \frac{I}{K} - a \right)^2. \tag{8f}$$

The transversality conditions are:

$$\lim_{t \rightarrow +\infty} \eta q e^{-\rho t} K = 0 \text{ and } \lim_{t \rightarrow +\infty} \mu e^{-\rho t} H = 0.$$

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<sup>5</sup>Defining this costate variable as  $\eta q e^{-\lambda t}$  rather than as a single variable is a matter of convenience, as it will become clear later on when we show that  $q$  plays a key role in determining investment.

Equation (8e) can be specified as

$$\frac{q-1}{h} + g = \frac{I}{K} = \frac{\dot{K}}{K}. \quad (9)$$

We can define  $P = \frac{\mu}{\eta}$  as the relative price of human capital in terms of goods. The rental rate of physical and human capital are represented, respectively, by  $r^K$  and  $r^H$ . Assuming there is incomplete specialization in production, the rental rate of each capital is a function of the relative price:

$$r^K = \alpha A \phi^{\alpha-1} P^{\frac{\alpha-1}{\alpha}} \quad (10a)$$

$$r^H = (1-\alpha) A \phi^\alpha P \quad (10b)$$

where  $\phi \equiv \left[ \frac{B}{A} (1-\alpha)^{-1} \right]^{\frac{1}{\alpha}}$  is a constant.

Notice that  $\eta = C^{-1} = U'(C)$ . It follows that  $\frac{\dot{P}}{P} = \frac{\dot{\mu}}{\mu} - \frac{\dot{\eta}}{\eta} = \frac{\dot{\mu}}{\mu} + \frac{\dot{C}}{C}$ . We also define  $c = \frac{C}{K}$ ,  $k = \frac{K}{H}$ ,  $y^K = \frac{Y^K}{K}$  and  $y^H = \frac{Y^H}{H}$ . Following Bond et al. (1996), we derive:

$$y^K \equiv \frac{Y^K}{K} = \frac{1}{\alpha} r^K \quad (11a)$$

$$y^H \equiv \frac{Y^H}{H} = B - \frac{1-\alpha}{\alpha} \frac{r^K}{P} k. \quad (11b)$$

The dynamic system is given by:

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \left( \frac{q-1}{h} + g \right) \quad (12a)$$

$$\frac{\dot{k}}{k} = \left( \frac{q-1}{h} + g \right) - y^H \quad (12b)$$

$$\dot{q} = \left( \rho + \frac{\dot{C}}{C} \right) q - r^K - (q-1) a - \frac{h}{2} \left( \frac{q-1}{h} \right)^2 \quad (12c)$$

$$\frac{\dot{P}}{P} = \left( \rho - \frac{r^H}{P} \right) + \frac{\dot{C}}{C} \quad (12d)$$

and it takes into account the transversality conditions for  $K$  and  $H$ .

It is enlightening to rearrange equation (12c) and substitute the expression  $\frac{\dot{C}}{C}$  in equation (12d):

$$\frac{\dot{P}}{P} = \tilde{r}^K - \frac{1}{P}r^H \quad (13)$$

where

$$\tilde{r}^K = \frac{r^K}{q} + \frac{\dot{q}}{q} + \frac{(q-1)a + \frac{h}{2}\left(\frac{q-1}{h}\right)^2}{q}. \quad (14)$$

We can analyze the transitional dynamics of the system by taking into account the behaviour of the relative price and the remuneration of the two capitals, following the no-arbitrage condition given by equation (13). Investing one unit of physical capital yields a net marginal product of physical capital in the consumption goods equal to  $\tilde{r}^K$ . On the other hand, investing  $\frac{1}{P}$  units of human capital yields  $\frac{\dot{P}}{P} + \frac{1}{P}r^H$ . In the balanced growth path,  $(\tilde{r}^K)^* = \left(\frac{1}{P}r^H\right)^* = r^*$ . We will call  $r^*$  the real interest rate in the balanced growth path.

The transitional path applies to any situation around the balanced growth path, when  $k_0 \neq k^*$ . For example, by assuming that  $k_0 > k^*$ , human capital is relatively less abundant and its remuneration will be relatively higher than physical capital:  $\frac{1}{P}r^H > \tilde{r}^K$ . Notice that there will be an incentive to invest relatively more in human capital. With the remuneration of the two capitals depending only on the relative price (hypothesis that applies when the adjustment costs go to zero) and taking into account equations (10b) and (10a),  $\frac{1}{P}r^H$  will be decreasing and  $\tilde{r}^K$  will be increasing during the transition to the balanced growth path. As human capital becomes relatively more available, the relative price will be decreasing.

We will return to these expressions of the transitional dynamics when comparing the closed and open economies.

### 3.2 The speed of convergence

Analyzing the case when human capital is produced only with human capital, it is possible to characterize the transitional dynamics of the model through a system of the two differential equations on  $q$  and  $P$ .

In this case, the ratios production of tradable goods over physical capital and consumption over physical capital depend only on  $q$  and  $P$ . Then we can analyze the dynamics of the system with the differential equations of Tobin's  $q$  and of the relative price  $P$ <sup>6</sup>. With  $r^H = PB$ , then  $(r^K)^* = B = \left(\frac{r^H}{P}\right)^*$ .

The two differential equations to be considered for the transitional dynamics are:

$$\dot{q} = \left(\rho + \frac{\dot{C}}{C}\right)q - r^K - (q-1)a - \frac{h}{2} \left(\frac{q-1}{h}\right)^2 \quad (12c)$$

$$\frac{\dot{P}}{P} = \left(\rho - \frac{r^H}{P}\right) + \frac{\dot{C}}{C} \quad (12d)$$

where

$$\frac{\dot{C}}{C} = \left(\frac{q-1}{h} + g\right) + \frac{1}{c} \left(\frac{1-\alpha-1}{\alpha} r^K \frac{\dot{P}}{P} - \frac{q}{h} \dot{q}\right). \quad (15)$$

We obtain equation (15) after replacing the expression  $\frac{\dot{c}}{c}$  of equation (12a), in two steps. First, we divide equation (5a) by  $K$ , noting that  $c = \frac{C}{K}$ . Second, we obtain a new expression for  $\frac{\dot{c}}{c}$  using the transformation of equation (5a) and equations (10a) and (11a).

An analytical solution for the speed of convergence of the linearized version of this model is derived and we can observe its behaviour when changing the parameter of adjustment costs. The negative eigenvalue of the linearized system is derived in Appendix A. We would like to point out that  $c$  does not appear in the linearized version of equations (12c) and (12d). Notice that the constant associated with changes in  $c$  in equation (15) is equal to zero, as in the balanced growth path  $\dot{P} = \dot{q} = 0$ .

Let  $\lambda_{C(h)} = -\mu$  be the speed of convergence of this closed economy with

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<sup>6</sup>In the general form where human capital is produced with both capitals, the linearized system has three equations depending on three variables. With such a system, it is not possible to find a clear analytical solution for the dynamics of the economy.

adjustment costs<sup>7</sup>:

$$2\lambda_{C(h)} = -\rho + \left[ \rho^2 + 4\frac{1-\alpha}{\alpha}B\Delta \right]^{\frac{1}{2}} \quad (16)$$

where

$$\Delta = \frac{1}{h}A_p^*A_q^* = \frac{1}{h \left( 1 + \frac{1}{c^*} \frac{1-\alpha}{\alpha} B \right) + \frac{1}{c^*}}.$$

One expects that by increasing the adjustment costs the speed of convergence will decrease, as in the Solow model with adjustment costs<sup>8</sup>. By assuming that human capital is produced only with human capital, Ortigueira and Santos (1997) make simulations of the speed of convergence for a closed economy with endogenous growth and adjustment costs for physical capital and find that by increasing the adjustment costs the speed of convergence decreases.

In Appendix A we also show that the speed of convergence of this closed economy is decreasing in the adjustment costs, that is  $\frac{\partial \lambda_{C(h)}}{\partial h} < 0$  and that the expressions  $A_p^*$  and  $A_q^*$  are functions of the parameters and of the values of the variables in the balanced growth path.

One would expect the speed of convergence of this economy to approach the speed of convergence of the closed economy without adjustment costs, when the adjustment costs go to zero. In fact:

$\lim_{h \rightarrow 0} \lambda_{C(h)} = \frac{1-\alpha}{\alpha}B$ , corresponding to the speed of convergence of the same model of a closed economy without adjustment costs. That is  $\lambda_C = \lambda_{C(h=0)}$ . This expression can be derived directly from equation (13) with  $\tilde{r}^K = r^K$ ,

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<sup>7</sup>The papers referred to above, Bond et al. (1996) and Mulligan and Sala-i-Martin (1993), do not arrive at an expression for the speed of convergence. Using the Lucas (1988) model, Ortigueira and Santos (1997) present an expression for the speed of convergence, but the analytical solution is derived in the appendix of their paper and only for the case without adjustment costs. These authors also analyze the behaviour of the speed of convergence of the non-linearized model and they show that the linearized model well represents the behaviour of the transitional dynamics over substantial phases of the transition. Working with the linearized version of the transitional dynamics, our results are all derived analytically.

<sup>8</sup>For the one sector model, we can derive an explicit solution for the speed of convergence of the linearized system. Abel and Blanchard (1983) characterize geometrically the dynamics of the one sector model with adjustment costs.

as  $q = 1$ . It follows  $\frac{\dot{P}}{P} = r^K - \frac{1}{P}r^H$ . Notice that we can also check the stable mechanism of the relative price, taking into account equations (10a) and (10b):  $r_P^K < 0$  and  $r_P^H > 0$ .

It is also interesting to know what happens if the adjustment costs go to infinite. We have

$$\lim_{h \rightarrow \infty} \lambda_{C(h)} = 0.$$

We would like to point out one conclusion that will be used later on when comparing the speed of convergence of the closed and the open economies. Since the adjustment costs have a finite value, the speed of convergence of the closed economy is always greater than zero.

## 4 The open economy to capital mobility

In this Section, we develop an economy open to capital mobility, where physical capital and consumption will be tradable goods and human capital will be assumed nontradable. There is adjustment costs for physical capital. The speed of convergence of the open economy is smaller than the speed of convergence of the closed economy. This result is in contrast with the dynamics of the one sector open economy with, or without, adjustment costs<sup>9</sup>.

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<sup>9</sup>The Solow model predicts a finite speed of convergence for a closed economy. Although, a small open economy with the same characteristics would have an infinite speed of convergence. With perfect international capital mobility, the remuneration of capital in a small open economy must be equal to the international interest rate and capital flows would eliminate any difference in remunerations instantaneously. Even allowing for adjustment costs in the small open economy, the closed economy with the same structure has a smaller speed of convergence. Credit constraints can be introduced in an economy using physical and human capital in production, where only physical capital can be used as collateral for foreign borrowing. The previous conclusion does not change and the values of the speed of convergence in the model may be closer to the values one finds in the empirical literature. For a discussion on convergence of open economies see Barro and Sala-i-Martin (1995), chapter 3, and Obstfeld and Rogoff (1996), chapter 7.

## 4.1 The model and the transitional dynamics

The planning problem of the open economy is given by

$$\max_{C,u,I,K,H,D} \int_0^{+\infty} (\log C) e^{-\rho t} dt \quad (17)$$

subject to

$$\dot{D} = r^* D + AK^\alpha (uH)^{1-\alpha} - C - I \left[ 1 + m \left( \frac{I}{K} \right) \right] \quad (18a)$$

$$\dot{K} = I \quad (18b)$$

$$\dot{H} = B [(1-u)H] \quad (18c)$$

with  $D_0$ ,  $K_0$  and  $H_0$  given.

Let  $D$  represent net foreign bonds accumulated by the economy and  $r^*$  be the international interest rate. Notice that equation (18a) replaces equation (5a) of the closed economy. The expression of the adjustment costs in equation (18a) is equal to the expression in equation (6) presented in the closed economy. We are assuming that  $r^*$  is equal to  $(r^K)^*$ , as determined in the balanced growth path of the closed economy. This implies that the closed and the open economy have the same production structure, as stated above, and that we are assuming the rest of the world is in the balanced growth path. Also notice that both economies have the same growth rate in the balanced growth path, as  $g = (r^K)^* - \rho = r^* - \rho$ .

The Hamiltonian for this problem is now

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t} \left\{ r^* D + AK^\alpha (uH)^{1-\alpha} - C - I \left[ 1 + m \left( \frac{I}{K} \right) \right] \right\} + \eta q e^{-\rho t} I + \mu e^{-\rho t} \{ B [(1-u)H] \}$$

where  $\eta$ ,  $\eta q$  and  $\mu$  are the costate variables in net foreign bonds, installed physical capital and human capital, respectively.

In addition to equations (8a) to (8e) of the closed economy, we obtain the

following first order conditions:

$$\dot{\eta} = \eta(\rho - r^*) \quad (19a)$$

$$\dot{q} = r^*q - r^K - h \left( \frac{I}{K} - a \right) \frac{I}{K} + \frac{h}{2} \left( \frac{I}{K} - a \right)^2. \quad (19b)$$

The transversality conditions are now:

$$\lim_{t \rightarrow +\infty} \eta q e^{-\rho t} K = 0, \quad \lim_{t \rightarrow +\infty} \mu e^{-\rho t} H = 0 \quad \text{and} \quad \lim_{t \rightarrow +\infty} \eta e^{-\rho t} D = 0.$$

As in the closed economy we have  $P = \frac{\mu}{\eta}$ , giving the relative price of human capital in terms of goods. In this open economy  $P$  is also the relative price of tradables over nontradables, that is a real exchange rate. As stated above,  $\eta$  is the value of foreign bonds. Remember that the costate variable of installed physical capital in the open economy is also  $\eta q$ , due to the adjustment costs, as in the closed economy.

The dynamic system of this open economy is given by the following equations:

$$\frac{\dot{c}}{c} = (r^* - \rho) - \left( \frac{q-1}{h} + g \right) \quad (20a)$$

$$\dot{d} = \left[ r^* - \left( \frac{q-1}{h} + g \right) \right] d + y^K - c - \left( \frac{q-1}{h} + a \right) - \frac{h}{2} \left( \frac{q-1}{h} \right)^2 \quad (20b)$$

$$\frac{\dot{k}}{k} = \left( \frac{q-1}{h} + g \right) - y^H \quad (20c)$$

$$\dot{q} = r^*q - r^K - (q-1)a - \frac{h}{2} \left( \frac{q-1}{h} \right)^2 \quad (20d)$$

$$\frac{\dot{P}}{P} = r^* - \frac{r^H}{P} \quad (20e)$$

and it takes into account the transversality conditions for  $D$ ,  $K$  and  $H$ . We are representing  $d = \frac{D}{K}$ , implying  $\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{K}}{K}$ .

The value of the relative price in the balanced growth path is given by equation (20e). Note that  $q^* = 1$ . With equation (20c) we have  $k^*$ . As we will see below,  $d^*$  and  $c^*$  depend on the initial conditions.

From the flow budget constraint, equation (18a), and by taking into account a no-Ponzi-game condition, we obtain the intertemporal budget constraint:

$$\int_0^{+\infty} C e^{-r^* t} = \int_0^{+\infty} \left\{ AK^\alpha (uH)^{1-\alpha} - I \left[ 1 + m \left( \frac{I}{K} \right) \right] \right\} e^{-r^* t} + D_0$$

where  $C_t = C_0 e^{(r^* - \rho)t}$ .

Before proceeding to next subsection, we make a reference to the dynamics of foreign bonds. The details of the solution of equation (20b), linearized around the balanced growth path, are in Appendix B. We have then

$$(d - d^*) = \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) e^{-\lambda_O t}$$

where the signs of the expressions in  $\Lambda$  are:  $y_P^K < 0$ ;  $y_k^K > 0$ ;  $\Pi > 0$ ;  $\Gamma < 0$ . It follows that the ambiguity of the sign of  $\Lambda$  comes from  $y_P^K$  being negative, as all other expressions are positive.

The ratio net foreign bonds over physical capital,  $d$ , is driven by the capital ratio,  $k$ . Because of the ambiguity of the sign of  $\Lambda$ , we have an ambiguity with the evolution of  $d$ . This ambiguity would disappear in the case  $\alpha < \beta$ , where the relative price has no transitional dynamics and the only term in  $\Lambda$  is  $y_k^K < 0$ .

Notice that we know  $d^*$  with this last equation, given  $t = 0$  and a  $k_0$ . Then one obtains  $c^*$  with equation (20b).

## 4.2 The speed of convergence

We now present some intuition on the transitional dynamics of the relative price in the open economy. As in Section 3 with equation (13) for the closed economy, relevant information for the open economy is provided by equation (20e). The incentive to accumulate relatively more one of the capitals depends on their remuneration. Consider again the case  $k_0 > k^{*10}$ . Human capital is relatively less abundant and its remuneration will be relatively

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<sup>10</sup>The conclusions do not change if we instead consider the case  $k_0 < k^*$ .

higher than physical capital:  $\frac{1}{P}r^H > r^*$ . Notice that now there is an alternative of investment given by the international interest rate  $r^*$ , which is fixed during the transition. However, the remuneration of physical capital changes during the transition in the closed economy, where  $r^K < r^*$  and  $\tilde{r}^K < r^*$ . This differentiated pattern in the closed and the open economies leads to a gap between remunerations which is smaller in the open economy. Consequently, the incentive to invest relatively more in human capital is higher in the closed economy and this economy will converge faster to the balanced growth path than an open economy. Using a two-sector endogenous growth model, à la Lucas (1988) and with adjustment costs, the speed of convergence is smaller in the open economy than in the closed economy.

The expression of the speed of convergence of the open economy,  $\lambda_O$ , is determined only with the differential equation of the relative price, as the remuneration of capital only depends on the relative price. See equation (20e). Given the structure of production in Lucas (1988), where the share of physical capital in the production of human capital is zero, the speed of convergence is (or goes to) zero in this open economy,  $\lambda_O \rightarrow 0$ .

The expression of the speed of convergence of the closed economy, given by equation (19c), is positive and thus  $\lambda_C > 0$ . Consequently  $\lambda_C > \lambda_O$ .

The main result of the paper is not specific to the case where the speed of convergence of the open economy goes to zero, as we will see in the next Section.

## 5 An extension of the model and a numerical example

The result presented above is not due to the specification used in Lucas (1988), which also leads to a simplification and clarity of exposition of the mechanism at work in the model.

Now we present an extension of the model and numerical simulations for different values of  $\beta$ , the share of physical capital in the production of human capital. Notice that with  $\beta > 0$ , the system of differential equations for

deriving the speed of convergence in the closed economy has three equations with three variables, implying that we do not have an analytical solution. But the speed of convergence of the open economy is again given by one differential equation on the relative price, as we generalize the case where the production of goods remains more intensive in physical capital ( $\alpha > \beta$ ). See again equation (20e).

Notwithstanding, the intuition presented above for the difference of the speed of convergence still applies. We rely on numerical simulations when comparing both economies with adjustment costs and with a positive share of physical capital in the production of human capital,  $\beta > 0$ .

With  $\alpha > \beta > 0$ , we have

$$Y^K = A(vK)^\alpha (uH)^{1-\alpha} \quad (21a)$$

$$Y^H = B[(1-v)K]^\beta [(1-u)H]^{1-\beta} \quad (21b)$$

where  $v$  ( $0 \leq v \leq 1$ ) is the fraction of the physical capital used in the consumption sector.

The rental rate of physical and human capital are again represented, respectively, by  $r^K$  and  $r^H$ . Assuming there is incomplete specialization in production, the rental rate of each capital also is a function of the relative price:

$$r^K = \alpha A \bar{\phi}^{\alpha-1} P^{\frac{\alpha-1}{\alpha-\beta}} \quad (22a)$$

$$r^H = (1-\alpha) A \bar{\phi}^\alpha P^{\frac{\alpha}{\alpha-\beta}} \quad (22b)$$

where  $\bar{\phi} \equiv \left[ \frac{B}{A} \left( \frac{\beta}{\alpha} \right)^\beta \left( \frac{1-\alpha}{1-\beta} \right)^{\beta-1} \right]^{\frac{1}{\alpha-\beta}}$  is a constant.

Taking into account equations (22a)-(22b) and equation (20e), the speed of convergence of the open economy is now equal to

$$\lambda_O = \frac{\beta}{\alpha - \beta} (r^K)^* . \quad (23)$$

The marginal productivity of physical capital is not restricted to be equal to the international interest rate during the transition, due to adjustment costs.

The stable mechanism of the relative price works, as described in Section 3. With equation (20d), we have  $r^K \neq r^*$ . With equation (20e), we have the gradual adjustment of the relative price giving also the transitional dynamics of the open economy. The stable mechanism of the relative price in the open economy works for any positive value of the adjustment costs. However, this stable mechanism could not work without adjustment costs. The reason is that the open economy with  $h = 0$  implies  $r^K = r^*$  and  $P = P^*$ , as  $r^K$  is only a function of  $P$ .

Before presenting the numerical simulations for the speed of convergence of the closed economy with adjustment costs and a positive share of physical capital in the production of human capital, we will consider an intuitive example. To a first approximation, the speed of convergence of the closed economy for a small value of the adjustment costs can be represented by the speed of convergence without adjustment costs. With this assumption, we have the following analytical expression:

$$\lambda_C = \frac{(1 - \alpha) + \beta}{\alpha - \beta} (r^K)^*. \quad (24)$$

Notice that with this assumption,  $q = 1$  and equation (13) becomes  $\frac{\dot{P}}{P} = r^K - \frac{r^H}{P}$ . See also the discussion in Section 3.

These expressions make it clear that the speed of convergence is higher for the closed economy. Thus, this result is not specific to human capital being produced only with human capital ( $\beta = 0$ ), in which case the speed of convergence goes to zero in the open economy. In fact the expression of  $\lambda_O$  also makes it clear that the speed tends to zero as  $\beta$  tends to zero. Notice that  $\lambda_C = \frac{(1-\alpha)}{\alpha} B$ , when  $\beta = 0$ , as in Section 3.

The difference between  $\lambda_O$  and  $\lambda_C$  is given by the expression  $(1 - \alpha)$  in  $\lambda_C$ . As far as  $0 < \alpha < 1$ , the remuneration of physical capital will be changing during the transitional path in the closed economy. In these conditions, the gap between remunerations will be higher in the closed economy than in the open economy. Only if  $\alpha = 1$  would this effect disappear, as  $(1 - \alpha) = 0$ .

As stated above, this first approximation of the speed of convergence of the closed economy with a small value of the adjustment costs is a specific

case. Do the conclusions on the speed of convergence change if the parameter of the adjustment costs is beyond a certain value?

To explore by how much the speed of convergence of the closed economy is affected by the adjustment costs, we now carry out numerical simulations.

The values of the parameters of the model we use are the following<sup>11</sup>:

- growth rate in the balanced growth path :  $g = 0.02$
- rate of depreciation:  $\delta_K = \delta_H = 0.05$
- subjective preference rate:  $\rho = 0.03$
- (inverse) intertemporal elasticity of substitution:  $\sigma = 1.5$
- share of physical capital in production of goods:  $\alpha = 0.4$

With these assumptions, the real interest rate in the balanced growth path is equal to  $r^* = 0.06$ . Using  $\beta = 0.1$ , we set  $B = 0.11$  and  $A = 1.516$ .  $A$  is adjusted to maintain the values of the real interest rate and the growth rate in the balanced growth path. Notice that using  $\beta = 0$  and taking into account equation (13) in the balanced growth path,  $B = (\frac{1}{P}r^H)^* = (r^K)^* = r^* + \delta = 0.11$ <sup>12</sup>.

All the results are presented in Table 1. Remember that the expressions for  $\lambda_C(h = 0)$  and for  $\lambda_O$  are given by equations (24) and (23), respectively. We simulate the values of  $\lambda_C$  for  $h > 0$  with MATLAB.

In the general case, we set  $\beta = 0.1$ . We then consider different values for the adjustment costs, following the values presented in Ortigueira and Santos (1997). We confirm our analytical result with all those values considered for the adjustment costs<sup>13</sup>. That is, the value of  $\lambda_C$  is greater than that of  $\lambda_O$ .

As we have seen analytically with  $\beta = 0$ , the speed of convergence is always higher in the closed economy than in the open economy. We also

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<sup>11</sup>For a derivation of the general model, see Barro and Sala-i-Martin (1995) and Bond et al. (1996). These values of the parameters are in line with the literature. In the general model Barro and Sala-i-Martin (1995) use  $\beta < 0.4$ . Most of the literature uses  $\beta = 0$ , following Lucas (1988), as in Ortigueira and Santos (1997).

<sup>12</sup>In this last case  $A$  does not affect  $(r^K)^*$ . But things are different with  $\beta > 0$ .

<sup>13</sup>In the paper of Ortigueira and Santos (1997) leisure enters the utility function and has, together with  $h$ , an effect on the speed of convergence.

confirm this result for the range of values considered for the adjustment costs.

[INSERT TABLE 1 ABOUT HERE]

After presenting the intuition of the main result of the paper in the first step of this discussion, we have shown our main conclusion using a general two-sector endogenous growth model with adjustment costs. We have shown this result both analytically and with numerical simulations. Within this framework, the results for the speed of convergence of closed and open economies are opposite to the results one obtains using the one sector neoclassical growth model in the same experiment.

## 6 Conclusion

We developed an endogenous growth model à la Lucas (1988) with adjustment costs to compare the speed of convergence of open and closed economies. As human capital was nontradable in the open economy, partial capital mobility was the only difference between these two economies. It was possible to analyze the transitional dynamics of the model with a no-arbitrage condition. The difference between remuneration of the capitals gives the incentive to accumulate relatively more in one of the capitals. The main implication of the model was a smaller speed of convergence in the open economy than in the closed economy. This result reverses the conclusions of the same experiment when using the one sector neoclassical growth model in the same experiment.

In terms of policy implications, our study suggests the need for more reflection on the idea that capital mobility will accelerate the convergence of economies to the balanced growth path, and thus will increase growth. Given the implication of the model, it would be of interest to empirically analyze these issues. Although the empirical literature is looking for proxies of financial openness and its effects on growth through alternative specifications, it has not analyzed in a systematic way the effects of capital mobility on the

speed of convergence. Further empirical studies on this issue seem to be an interesting topic for future research.

## Appendix A The negative eigenvalue associated with the closed economy

Here we derive the negative eigenvalue associated with the linearization of the system given by equations (12c), (12d) and (15) in Section 3.

When human capital is produced only with human capital, the system is analyzed with the two differential equations on  $q$  and  $P$ . As presented in the text, the system is given by:

$$\dot{q} = \left( \rho + \frac{\dot{C}}{C} \right) q - r^K - (q-1)a - \frac{h}{2} \left( \frac{q-1}{h} \right)^2$$

$$\frac{\dot{P}}{P} = \left( \rho - \frac{r^H}{P} \right) + \frac{\dot{C}}{C}$$

where

$$\frac{\dot{C}}{C} = \left( \frac{q-1}{h} + g \right) + \frac{1}{c} \left( \frac{1-\alpha}{\alpha} \frac{1-\alpha}{\alpha} r^K \frac{\dot{P}}{P} - \frac{q}{h} \dot{q} \right).$$

We would like to point out that in the linearization of the equation above, the constant associated with changes in  $c$  is equal to zero as  $\dot{P}$  and  $\dot{q}$  are also equal to zero in the balanced growth path.

Reorganizing terms leads to

$$\dot{q} = A_q \left[ \rho q - r^K - (q-1)a - \frac{h}{2} \left( \frac{q-1}{h} \right)^2 + q A_P \frac{q-1}{h} + qg \right]$$

$$\frac{\dot{P}}{P} = A_P \left[ (\rho - B) + \left( \frac{q-1}{h} + g \right) - \frac{1}{c} \frac{q}{h} \dot{q} \right]$$

where

$$A_P = \frac{1}{1 + \frac{1}{c} \frac{1-\alpha}{\alpha} r^K}$$

and

$$A_q = \frac{1}{1 + q A_P \frac{1}{c} \frac{q}{h}}.$$

The linearized system can be represented by:

$$\begin{bmatrix} \dot{q} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} * \begin{bmatrix} q - 1 \\ P - P^* \end{bmatrix}$$

where

$$\begin{aligned} a_{11} &= \left[ \rho + A_P^* \frac{1}{h} \right] A_q^* \\ a_{12} &= \frac{1 - \alpha}{\alpha} \left( \frac{r^K}{P} \right)^* A_q^* \\ a_{21} &= A_P^* P^* \left[ \frac{1}{h} - \frac{1}{c^*} \frac{1}{h} A_q^* \left( \rho + A_P^* \frac{1}{h} \right) \right] \\ a_{22} &= -A_P^* A_q^* \frac{1}{c^*} \frac{1}{h} \frac{1 - \alpha}{\alpha} (r^K)^* . \end{aligned}$$

Notice that  $A_P^*$  and  $A_q^*$  correspond to the expression of  $A_P$  and  $A_q$ , but are evaluated with the values of the variables of the model in the balanced growth path.

With  $a = g = B - \rho$  and using equation (10a) one can show that  $c^* = \frac{(r^K)^*}{\alpha} - g = \frac{1 - \alpha}{\alpha} B - \rho$ . It follows that:

- the trace is equal to  $\text{Tr} = \rho$
- the determinant equal to  $\text{Det} = -\frac{1 - \alpha}{\alpha} B \frac{1}{h} A_P^* A_q^*$
- and the negative eigenvalue equal to  $2\mu = \rho - \sqrt{\rho^2 + 4 \frac{1 - \alpha}{\alpha} B \frac{1}{h} A_P^* A_q^*}$ .

Let  $\lambda_{C(h)} = -\mu$  be the speed of convergence of this closed economy with adjustment costs. By rearranging terms, we can represent the analytical solution for the speed of convergence as

$$2\lambda_{C(h)} = -\rho + \left[ \rho^2 + 4 \frac{1 - \alpha}{\alpha} B \Delta \right]^{\frac{1}{2}} \quad (\text{A1})$$

where

$$\Delta = \frac{1}{h} A_P^* A_q^* = \frac{1}{h \left( 1 + \frac{1}{c^*} \frac{1 - \alpha}{\alpha} B \right) + \frac{1}{c^*}} .$$

The expression of the speed of convergence corresponds to equation (23) in the text.

The speed of convergence of this closed economy is decreasing in the

adjustment costs:

$$\begin{aligned} \frac{\partial \lambda_{C(h)}}{\partial h} &= \frac{1}{2} \left[ \rho^2 + 4 \frac{1-\alpha}{\alpha} B \Delta \right]^{\frac{1}{2}-1} * \\ &* 4 \frac{1-\alpha}{\alpha} B \Delta^2 \left( 1 + \frac{1}{c^*} \frac{1-\alpha}{\alpha} B \right) (-1) < 0. \end{aligned} \quad (\text{A2})$$

## Appendix B The dynamics of foreign bonds

This appendix derives the dynamics of foreign bonds presented in Section 4.

We can now analyze the dynamics of consumption and of net foreign bonds. Linearizing equation (20a) we have

$$\dot{c} = -\frac{c^*}{h} (q - 1) = -\frac{c^*}{h} \Gamma (k_0 - k^*) e^{-\lambda_O t}$$

and the solution of this differential equation is given by

$$(c - c^*) = \frac{-\frac{c^*}{h} \Gamma}{-\lambda_O} (k_0 - k^*) e^{-\lambda_O t}.$$

By linearizing equation (20b) and by taking into account the previous equations:

$$\dot{d} = (r^* - g) (d - d^*) + \Lambda (k - k^*)$$

with

$$\Lambda = \left[ (y_P^K \Pi + y_k^K) + \frac{c^* \Gamma}{-\lambda_O} - \left( \frac{d^* + 1}{h} \right) \Gamma \right]$$

and

$$\Gamma = \frac{-(\lambda_O + a_{11})}{\frac{k^*}{h} - \frac{a_{13}}{a_{23}} (r^* - g + \lambda_O)} \quad \text{and} \quad \Pi = \frac{-(r^* - g + \lambda_O)}{a_{23}} \Gamma.$$

The signs of the expressions in  $\Lambda$  are:  $y_P^K < 0$ ;  $y_k^K > 0$ ;  $\Pi > 0$ ;  $\Gamma < 0$ . These expressions are related to the solution of the system of differential equations,  $k$ ,  $q$  and  $P$ . It follows that the ambiguity of the sign of  $\Lambda$  comes from  $y_P^K$  being negative, as all other expressions are positive.

The solution of this differential equation is given by:

$$(d - d^*) = \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) e^{-\lambda_O t} + \left[ d_0 - d^* - \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) \right] e^{(r^* - g)t} \quad (\text{A3})$$

with the equation in brackets set equal to zero to observe the transversality condition of net foreign bonds,  $D$ . We have then

$$(d - d^*) = \frac{\Lambda}{-\lambda_O - (r^* - g)} (k_0 - k^*) e^{-\lambda_O t}. \quad (\text{A4})$$

This is the equation presented in the text.

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Table 1: Comparing the speed of convergence of closed and open economies

	$\lambda_c$					$\lambda_o$
	$h = 0$	$h = 5$	$h = 10$	$h = 16$	$h = 32$	$h > 0$
$\beta = 0.1$	<b>0.25(6)</b>	0.146	0.122	0.109	0.094	<b>0.03(6)</b>
$\beta = 0$	<b>0.165</b>	0.118	0.103	0.095	0.085	<b>0</b>

Notes:  $\lambda_c$  and  $\lambda_o$  represent the speed of convergence of the closed and open economy, respectively.  $h$  is the adjustment cost and  $\beta$  is the share of physical capital in the production of human capital. See the text for the values of other parameters.